

Real-Time Demand Response Model

Antonio J. Conejo, *Fellow, IEEE*, Juan M. Morales, *Student Member, IEEE*, and Luis Baringo, *Student Member, IEEE*

Abstract—This paper describes an optimization model to adjust the hourly load level of a given consumer in response to hourly electricity prices. The objective of the model is to maximize the utility of the consumer subject to a minimum daily energy-consumption level, maximum and minimum hourly load levels, and ramping limits on such load levels. Price uncertainty is modeled through robust optimization techniques. The model materializes into a simple linear programming algorithm that can be easily integrated in the Energy Management System of a household or a small business. A simple bidirectional communication device between the power supplier and the consumer enables the implementation of the proposed model. Numerical simulations illustrating the interest of the proposed model are provided.

Index Terms—Bidirectional communication, demand response, hourly prices, optimization, smart grids.

NOMENCLATURE

The main notation used throughout the paper is stated below for quick reference. Other symbols are defined as required throughout the text.

d_k	Consumer demand at the beginning of hour k .
e_{day}	Minimum daily consumption required by the consumer.
e_k	Energy consumption in hour k .
r^U/r^D	Up/down demand ramping limit.
$u_k(\cdot)$	Consumer utility in hour k .
λ_k	Energy price in hour k .

A superscript $a/\text{min}/\text{max}$ affecting any of the symbols above indicates actual/minimum/maximum value.

I. INTRODUCTION

A. Motivation and Technique

THE STILL VAGUE concept of *smart grids* refers to those electricity networks equipped with the technology required to facilitate the fluent interaction of all users connected to it [1]. Among other things, smart grid technology enables bidirectional communication between a power supplier and

different types of consumers. Such bidirectional communication may be used by a conscious consumer to optimize its energy consumption profile so that its utility is maximized (or its electricity cost minimized).

The contractual arrangement between the consumer and the supplier allows the consumer to receive hourly price information several minutes prior to the corresponding hour and responds to this information by adjusting its consumption for that hour. Such hourly energy adjustment is made within a daily planning framework so that a minimum daily energy consumption is guaranteed. Other consumption constraints, such as hourly load and load-variation limits, are also enforced.

Specifically, we consider a daily 24-h horizon spanning the $t - 1$ hours prior to the current one, the current hour t , and the $24 - t$ hours following the current one.

It is important to note that:

- 1) Prices and decisions (hourly energy consumptions and demand levels) for the initial $t - 1$ hours are known.
- 2) The price for the current hour t is known (e.g., 10 min prior to the current hour) and also the consumer demand at the beginning of this hour t , but not its energy consumption, which needs to be determined and communicated to the electricity supplier (e.g., 5 min prior to the current hour).
- 3) The prices for the following $24 - t$ hours are unknown data whose uncertainty needs to be modeled.
- 4) The preliminary energy consumption levels for the following $24 - t$ are variables to be determined.

Observe that the real-time availability of electricity price information, thanks to smart grid technology, is what makes the proposed demand response model useful from a practical point of view.

The price uncertainty pertaining to the $24 - t$ hours following the current one is considered via robust optimization, because robust optimization is particularly suited to address uncertain, but bounded parameters [3], [4]. The hourly price uncertainty is modeled using a forecast value and a certainty interval around such forecast value, e.g., $\lambda_t^{\text{exp}} \in [\lambda_t^{\text{min}}, \lambda_t^{\text{max}}]$. Conventional forecasting techniques are used to compute such confidence interval [5], [6].

For hourly decision making we consider the planning horizon described above that allows computing consumption levels by the consumer for the current hour and the following $24 - t$ ones. However, only the consumption of the current hour is actually “used” and sent to the energy supplier as the actual consumption in that hour. In other words, a rolling window model is used on a hourly basis to derive the optimal consumption for the current hour to be transmitted to the supplier.

The considered model translates into a simple linear programming problem that can be solved in virtually no time

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The authors are with the University Castilla-La Mancha, Ciudad Real, Spain, (e-mail: Antonio.Conejo@uclm.es; JuanMiguel.Morales@uclm.es; Luis.Baringo@uclm.es).

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using current PC computational technology. Moreover, the corresponding solution algorithm is easily embedded into the Energy Management System (EMS) of any household or small business.

The implementation of the decisions obtained using the proposed model generally results in significant energy cost savings, which is illustrated in this paper via different case studies.

B. Literature Review and Contribution

Much has been said and written about the smart grids' promising capabilities (e.g., demand-side management [7], [8], vehicle-to-grid systems [9], [10], and enhanced control of generation [11]) and their associated benefits (e.g., enabling infrastructure for integrating large amounts of renewable energy and installing distributed generation [12]–[14], new energy services, and network-usage and energy efficiency improvements [11], [15], [16]). But the fact is that much more has to be done in practice so as to turn smart grids into a reality.

Additionally, the technical literature includes a significant number of references dealing with the problem of a consumer sufficiently large to participate in the electricity markets to minimize its energy procurement costs (see, for instance, [17]–[19] and the references therein). For the time being, however, small consumers merely engage in fixed-price contracts with retailers. The role of a retailer basically consists in purchasing energy from the electricity markets to resell it to their clients at a price as competitive as possible, thus assuming the risk associated with market prices [20], [21].

Yet, with the development of smart grids, the interaction between a consumer and its power supplier is expected to become more *involved* due to the availability of technology enabling new forms of contractual agreements; and, in turn, with the *upgrade* of commercial relationships in the electricity sector, the design of new decision-making models are to be imperative. One of these new forms of agreements is the *real-time pricing* of electricity (RTP) [22], [23] according to which retail electric prices change frequently to reflect variations in the cost of the energy supply.

The contribution of this paper is in line with this RTP scheme and consists in providing a simple optimization model that allows a consumer to adapt its hourly load level in response to hourly electricity prices. Two-way communication between the consumer and the supplier is considered. This algorithm can be easily embedded into the EMS of a household or a small business and make it possible to achieve maximum utility by the consumer. To the best of our knowledge, no similar real-time demand response model under uncertainty has been proposed in the technical literature.

C. Paper Organization

The rest of the paper is organized as follows. Section II describes the proposed model including an initial model and a *robust* one. Section III explains the implementation of the model. Section IV provides and discusses results from a case study. Section V closes giving some relevant conclusions.

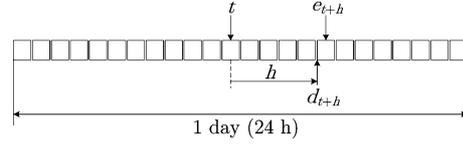


Fig. 1. Roles of time indexes t and h in the optimization model (1).

II. DEMAND RESPONSE MODEL

A. Initial Model

The demand response model is formulated below:

$$\text{Minimize}_{\{e_{t+h}, h=0, \dots, 24-t\} \cup \{d_{t+h}, h=1, \dots, 25-t\}} \lambda_t^a e_t - u_t(e_t) + \sum_{h=1}^{24-t} [\{\lambda_{t+h}\} e_{t+h} - u_{t+h}(e_{t+h})] \quad (1a)$$

subject to:

$$\sum_{h=1}^{t-1} e_h^a + e_t + \sum_{h=1}^{24-t} e_{t+h} \geq e_{\text{day}} \quad (1b)$$

$$e_{t+h} = \frac{d_{t+h} + d_{t+h+1}}{2}, \quad h = 0, \dots, 24-t \quad (1c)$$

$$d_{t+h} - d_{t+h+1} \leq r^D, \quad h = 0, \dots, 24-t \quad (1d)$$

$$d_{t+h+1} - d_{t+h} \leq r^U, \quad h = 0, \dots, 24-t \quad (1e)$$

$$d_{t+h+1}^{\min} \leq d_{t+h+1} \leq d_{t+h+1}^{\max}, \quad h = 0, \dots, 24-t. \quad (1f)$$

Model (1) is defined for each of the 24 hours of a day. Note that in each hour t , prices and hourly energy consumptions for the previous $t-1$ are known, while the energy consumptions ($e_{t+h}, h = 0, \dots, 24-t$) and the load levels ($d_{t+h}, h = 1, \dots, 25-t$) for the current hour t and the following $24-t$ ones are the variables to be determined.

The objective function (1a) to be minimized is the minus utility of the consumer spanning the current hour and the following $24-t$ hours. The $t-1$ hours prior to the current one are not considered in the objective function, because the utility of the consumer spanning that period is a known constant. Note that the price at hour t , λ_t^a , is known, but prices for the following $24-t$ hours, $\{\lambda_{t+h}\}, h = 1, \dots, 24-t$, are unknown, which is indicated by the brackets. Constraint (1b) establishes a floor for the daily consumption. That is, this constraint guarantees a minimum energy consumption per day. Constraints (1c) relate power and energy in each hour using a trapezoidal criterion. Constraints (1d) and (1e) are ramping down/up limits on hourly load levels. Constraints (1f) enforce bounds (min and max) on hourly load levels.

Fig. 1 clarifies graphically the roles of time indexes t and h in the optimization problem (1).

B. Robust Model

It should be noted that model (1) is not properly formulated as prices $\{\lambda_{t+h}\}, h = 1, \dots, 24-t$ are unknown quantities. We consider certainty intervals at the α -confidence level for prices,

$\lambda_{t+h}^{\min} \leq \lambda_{t+h} \leq \lambda_{t+h}^{\max}, h = 1, \dots, 24 - t$, and formulate the well-defined robust model below [3]:

Minimize

$$\lambda_t^a e_t - u_t(e_t) + \sum_{h=1}^{24-t} [\lambda_{t+h}^{\min} e_{t+h} - u_{t+h}(e_{t+h})] + \beta\Gamma + \sum_{h=1}^{24-t} \xi_{t+h} \quad (2a)$$

subject to:

$$\text{Constraints (1b)-(1f)} \quad (2b)$$

$$\beta + \xi_{t+h} \geq (\lambda_{t+h}^{\max} - \lambda_{t+h}^{\min}) y_{t+h}, \quad h = 1, \dots, 24 - t \quad (2c)$$

$$\xi_{t+h} \geq 0, \quad h = 1, \dots, 24 - t \quad (2d)$$

$$y_{t+h} \geq 0, \quad h = 1, \dots, 24 - t \quad (2e)$$

$$\beta \geq 0 \quad (2f)$$

$$e_{t+h} \leq y_{t+h}, \quad h = 1, \dots, 24 - t. \quad (2g)$$

The decision variables of robust problem (2) above are $\{e_{t+h}, h = 0, \dots, 24 - t\} \cup \{d_{t+h}, h = 1, \dots, 25 - t\} \cup \{\beta\} \cup \{\xi_{t+h}, h = 1, \dots, 24 - t\} \cup \{y_{t+h}, h = 1, \dots, 24 - t\}$.

Robust problem (2) is obtained using duality properties and exact linear equivalences. Variables β and ξ_{t+h} are dual variables of the initial problem (1) used to take into account the known bounds of prices, while y_{t+h} is an auxiliary variable used to obtain equivalent linear expressions. An exhaustive description of how to obtain this robust problem from problem (1) is given in [3].

Γ is a parameter that controls the level of robustness in the objective function. This parameter takes real values in the interval $[0, 24 - t]$, i.e., between zero and the number of unknown prices. This way, if $\Gamma = 0$ the influence of the price deviations in the objective function is ignored, while if $\Gamma = 24 - t$, all price deviations are considered, thus leading to a more conservative solution.

III. MODEL IMPLEMENTATION

Model (2) is to be implemented on a hourly basis using a rolling window criterion as follows:

- 1) The price for hour t , λ_t^a is transmitted by the supply company and received by the consumer prior (e.g., 10 minutes) to hour t .
- 2) Model (2) is solved to obtain the energy consumption in hour t , e_t^* , and the demand d_{t+1}^* at the beginning of hour $t+1$. These quantities are transmitted to the supply company prior (e.g., 5 minutes) to hour t .
- 3) The above two steps are repeated for the whole day on a hourly basis.

Note that the above procedure implies a cooperative agreement between the supplier and the consumer, as the supplier provides hourly prices and the consumer hourly consumptions and maximum demands.

The historical series of electricity prices up to and including hour t is used to build an ARIMA-based model from which confidence intervals for the energy prices in the subsequent $24 - t$ hours are estimated. These confidence intervals are, in turn, used as the uncertainty bounds, $\lambda_{t+h}^{\min} \leq \lambda_{t+h} \leq \lambda_{t+h}^{\max}, h =$

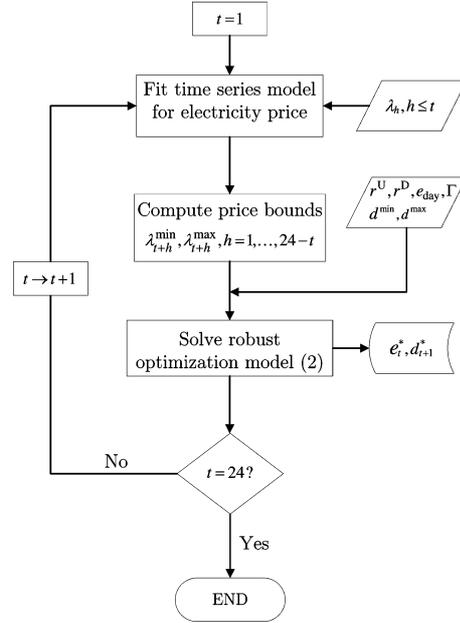


Fig. 2. Flow chart illustrating the rolling process.

$1, \dots, 24 - t$, in the robust optimization problem (2). These bounds are updated in each hourly period as new information is incorporated into the ARIMA fitting stage. Fig. 2 illustrates a flow chart of the consumption allocating process.

The values of parameters e_{day} , r^D and r^U are set beforehand by the consumer, and they can be reset as required during the rolling process to reflect changes in the energy consumption pattern or in consumer's elasticity.

In summary:

- 1) The considered consumer has a communication system that allows bidirectional communication with the power supplier on a hourly basis.
- 2) It receives each hourly price prior to the corresponding hour (e.g., 10 min ahead).
- 3) It has a price forecasting routine that provides price confidence intervals for the remaining hours of the day on a hourly basis. Alternatively, it receives such information from the power supplier.
- 4) It has an optimization routine to solve problem (2) to derive the optimal hourly consumption and optimal demand level to be sent to the power supplier prior to the corresponding hour (e.g., 5 min in advance).

It is important to emphasize that the smart grid technology provides the consumer with valuable information that the consumer can exploit to increase its utility. Roughly speaking, this increase can be attributed to the following.

- 1) The knowledge of the real-time electricity price, which enables the real-time adjustment of the energy consumption in accordance with the actual cost of electricity.
- 2) The availability of the price series up to the current point in time, which allows the consumer to update and improve its price uncertainty modeling in each hour.

The effects on the consumer utility of these two information management improvements are separately and jointly analyzed in the following case study.

TABLE I
CONSUMER DATA

Maximum hourly demand	3	MW
Minimum hourly demand	0	MW
Minimum daily consumption	15	MWh
Ramping up limit	1	MW/h
Ramping down limit	1	MW/h
Consumer utility	41.5	€/MWh

TABLE II
ENERGY PRICE DATA (€/MWh)

t	λ_t^a	λ_t^{\min}	λ_t^{\max}
1	44.80	39.00	52.44
2	41.03	33.36	49.90
3	36.10	27.64	44.93
4	33.00	24.05	41.24
5	33.00	22.59	39.80
6	36.46	23.25	41.76
7	43.01	27.73	50.40
8	47.05	30.77	56.29
9	46.06	30.75	56.51
10	45.51	31.53	58.11
11	46.06	32.61	60.20
12	44.50	32.06	59.26
13	45.61	32.83	60.72
14	45.42	32.47	60.08
15	39.28	31.58	58.45
16	41.16	32.41	59.98
17	42.01	32.38	59.95
18	43.00	32.28	59.77
19	41.16	31.65	58.60
20	41.63	30.74	56.91
21	42.00	29.67	54.94
22	41.16	29.29	54.24
23	41.87	30.18	55.87
24	36.81	28.83	53.41

IV. CASE STUDY

A. Data

Data for the considered commercial consumer is provided in Table I. The consumer demand at the beginning of the market horizon is 1.5 MW.

For the sake of simplicity the consumer utility is initially assumed to be constant throughout the hours of the day. However, this is not generally the case.

Price data is given in Table II. The second column provides the actual prices, while the third and fourth columns include the lower and upper bounds of prices for each hour, respectively. These data correspond to the energy prices of the Spanish area of the electricity market of the Iberian Peninsula on Monday July 5, 2010 [24]. Price bounds are obtained using the ARIMA model proposed in [6] with a level of confidence equal to 95%. Price bounds and actual prices are plotted in Fig. 3.

B. Results and Discussion

This subsection illustrates the working of the proposed method. For this purpose, problem (2) is solved using the data provided in the previous subsection.

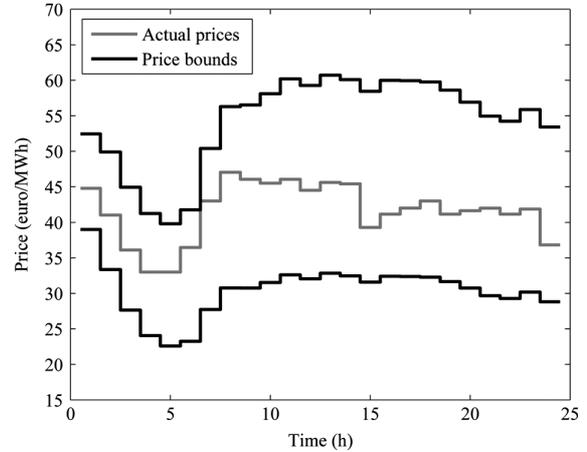


Fig. 3. Actual prices and price bounds.

In order to show the usefulness of the proposed algorithm, two alternatives are considered:

- 1) With smart grid. The problem is solved using the technique described in Section III.
- 2) Without smart grid. In this case, the consumer does not receive hourly price information. The only information available for the consumer are price bounds for the next day, so it has to determine its energy consumption profile for the whole day (with no hourly adjustments). This alternative boils down to solving problem (2) only for $t = 1$ with λ_1^a being unknown. Thus the objective function of problem (2) becomes

$$\sum_{h=1}^{24} [\lambda_h^{\min} e_h - u_h(e_h)] + \beta\Gamma + \sum_{h=1}^{24} \xi_h.$$

First of all, the optimal value of the control parameter Γ should be determined. Note that Γ takes values in the interval $[0, 24 - t]$. Table III shows the results for different values of the control parameter Γ as a percentage of its maximum value $(24 - t)$. The second and third columns provide the consumer utility with and without smart grid, respectively.

The model is solved using CPLEX 11.2.1 [25] under GAMS [26] on a Linux-based server with four processors clocking at 2.6 GHz and 32 GB of RAM. The optimal solution for each of the values of Γ is achieved in less than 1 s.

The optimal value of Γ , i.e., the value of Γ that results in the maximum daily utility for the consumer with smart grid, is 45% of the number of unknown prices, while without smart grid, values of Γ between 75% and 100% provide the same optimal daily utility for the consumer.

The use of the smart grid model allows achieving a daily utility for the consumer that is 15.86% higher than that obtained in the absence of smart grid. This is a consequence of the bidirectional communication and the hourly adjustments in the energy consumption, which is adapted to the hourly price profile.

Fig. 4 shows the energy consumption profile corresponding to the optimal value of Γ for the two considered alternatives. Note from Table I that the consumer utility function is linear, which means that the consumer obtains a positive profit provided that

TABLE III
DAILY CONSUMER UTILITY (€) FOR DIFFERENT VALUES OF THE CONTROL
PARAMETER Γ

Γ %	With smart grid	Without smart grid
0	-2.49	-2.49
5	-2.13	-2.22
10	-1.87	-1.88
15	-9.7	1.05
20	0.22	1.23
25	1.24	2.56
30	2.06	21.21
35	6.95	24.95
40	72.24	25.12
45	77.07	25.16
50	66.81	18.94
55	51.03	37.61
60	56.68	37.61
65	64.88	48.09
70	69.73	57.92
75	69.73	66.52
80	69.73	66.52
85	69.73	66.52
90	69.73	66.52
95	69.73	66.52
100	69.73	66.52

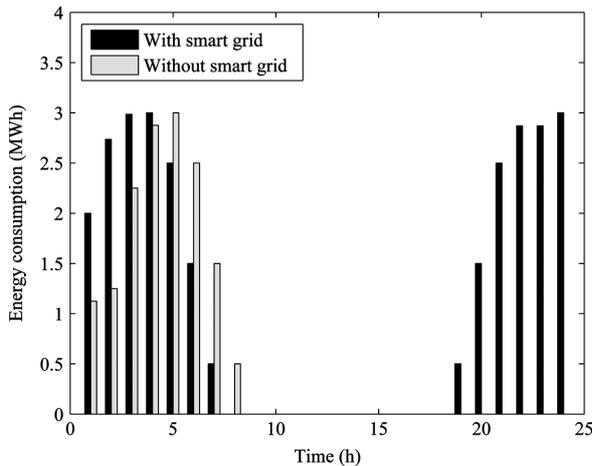


Fig. 4. Energy consumption profile for the optimal values of Γ .

the energy prices are lower than €41.5/MWh. Note also from Table II that between hours 15 and 24, there are several hours in which the energy prices are lower than €41.5/MWh. If the smart grid is used, the consumer gets this information and adapts its load profile accordingly, while if the smart grid is not available, the consumer cannot change its load profile (which must be pre-specified at the beginning of the day). This behavior is illustrated in Fig. 4.

Finally, we compare both alternatives (with and without smart grid) for the working days of the second week of July 2010 (i.e., from July 5th to 9th). The results obtained from this comparison are listed in Table IV. The second and third columns include the daily utility for the consumer with and without smart grid, respectively. The fourth column provides the utility increment

TABLE IV
RESULTS

Day	Utility (€)		Increment (%)	Optimal Γ (%)	
	SG	No SG		SG	No SG
Mon	77.07	66.52	15.86	45	70
Tue	169.12	131.91	28.21	35	80
Wed	128.34	118.04	8.72	45	80
Thu	119.99	112.84	6.34	40	80
Fri	79.99	78.20	2.28	75	85
Mean	114.90	101.50	13.20	-	-

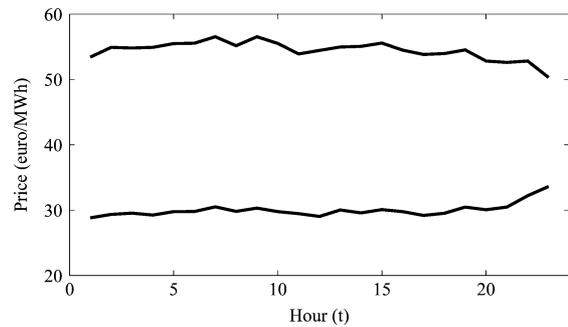


Fig. 5. Evolution throughout the day of the price bounds in hour 24.

achieved with the use of the smart grid. Lastly, the fifth and sixth columns provide the optimal values of Γ as a percentage of the unknown prices for each day and alternative.

Observe that using the smart grid, we obtain a weekly average utility that is 13.20% higher than in the case of not using it.

Note also that the optimal value of Γ for each alternative is different through the week due to its dependence with the energy prices and the consumer utility. However, this information is not available (we do not know the actual prices in advance), so a previous study is required to select the optimal value of this parameter for each period. Nevertheless, observe that from Monday to Thursday, the optimal value of Γ with smart grid is close to 40%. Likewise, without smart grid, and for all the working days of the week, values of Γ between 85% and 100% of the total unknown prices provide the same optimal solutions.

The results above have been obtained without updating the price bounds throughout the day, i.e., in each hour t the price bounds used to solve problem (2) were identical to those provided in Table II. Nevertheless, as the rolling process progresses new information about the time series of electricity prices is collected. Then, this information can be incorporated into the ARIMA model, thus updating the price bounds for the remaining hours of the day.

Fig. 5 depicts the evolution of the price bounds in hour 24 throughout the day. Note that the main changes in these bounds occur in the three hours before hour 24.

The effect of updating price bounds is shown in Table V. The third and second columns show the daily consumer utility for the price bounds being updated or not, respectively, while the fourth column provides the utility increment achieved updating such bounds.

As shown in Table V, the price bound updating does not always cause an increase in the utility for the consumer. This is so because of the following four reasons:

TABLE V
EFFECT OF UPDATING PRICE BOUNDS THROUGHOUT THE DAY

Day	Utility (€)		Increment (%)
	No price bound updating	Price bound updating	
Mon	77.07	72.63	-5.72
Tue	169.12	166.20	-1.73
Wed	128.34	135.10	5.27
Thu	119.99	122.02	1.69
Fri	79.99	79.99	0.00

- 1) The price bounds are barely affected by the updating process, except for those immediate hours following the current one, in which price bounds get a little bit closer.
- 2) The narrowing of the price confidence interval is more significant in the last few hours of the rolling process, while most of the energy consumption is scheduled for the first hours.
- 3) In each step of the rolling process, the solution provided by the robust model is mainly determined by the highest upper bounds of the hourly prices following the current hour. However, as stated before, these upper bounds are not substantially affected when updated.
- 4) The inaccuracies associated with the modeling of the electricity price series may well mask, or even reverse, the slight improvement achieved by the price bound update.

For all the reasons above, we should conclude in this case that the real-price availability is significantly much more important than updating price bounds. However, we emphasize that this might not be the case, for example, in situations with high consumption during the last hours of the day, for which price bounds noticeably approach in relative terms.

Next, in order to obtain more realistic results, the problem is solved using a nonconstant utility for the consumer. Thus, the consumer hourly utility function is modeled by four blocks of 75 MWh each, with a utility per block of €46, €43, €40, and €37/MWh, respectively.

The remaining data is equal to that provided in Section IV-A. The problem is solved for the following four cases:

- 1) Case #1: with smart grid and no price bound updating throughout the day.
- 2) Case #2: without smart grid.
- 3) Case #3: with smart grid and price bound updating.
- 4) Case #4: with smart grid and forecast prices. In this case, the initial problem (1), which is non-robust, is solved using forecast prices in the objective function, but no price bounds. That is, we solve problem (1) with the following objective function:

$$\lambda_t^a e_t - u_t(e_t) + \sum_{h=1}^{24-t} \left[\lambda_{t+h}^{\text{exp}} e_{t+h} - u_{t+h}(e_{t+h}) \right]$$

where $\lambda_{t+h}^{\text{exp}}$ is the forecast price in hour $t + h$, which is obtained using the ARIMA model proposed in [6].

The total consumer utilities for each case and working day of the second week of July 2010 are provided in Table VI. The results for the robust models have been obtained using a value

TABLE VI
RESULTS WITH A NONCONSTANT UTILITY FUNCTION

Day	Utility (€)			
	Case #1	Case #2	Case #3	Case #4
Mon	73.33	66.52	72.89	58.69
Tue	133.32	127.62	131.45	130.99
Wed	123.40	121.26	123.96	101.52
Thu	99.97	94.28	104.53	88.66
Fri	76.90	73.13	76.13	56.27

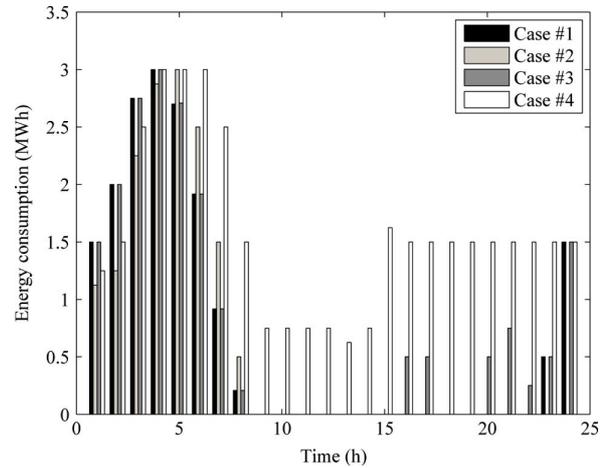


Fig. 6. Energy consumption profile.

of Γ equal to 85% of the unknown prices, which is not the optimal value for all the cases. Nevertheless, this information is not available in advance, and the results with this value of Γ are very close to the optimal ones.

As shown in Table VI, the use of smart grid technology increases the utility for the consumer. Particularly, the weekly average utility with smart grid is 4.99% higher than that obtained without smart grid.

With respect to updating price bounds throughout the day, the results are very similar to those obtained with no price bound updating.

Finally, note that the robust model proposed in this paper provides a better solution than that obtained using simpler techniques, as the one employed in case #4, which uses price forecasts to solve the problem. The weekly average utility is 16.22% higher with the robust model than that obtained with price forecasts.

The energy consumption profile on Monday for each case is depicted in Fig. 6.

V. CONCLUSION

This paper provides a simple LP algorithm to be integrated into the EMS of a household or a small business. Via bidirectional communication with the electricity supplier, such algorithm allows maximizing the consumer utility or minimizing its energy cost. The interaction takes place on a hourly basis using a rolling window algorithm to consider the energy consumption throughout the twenty four hours of the day. Bidirectional communication is a key component of a smart grid

and as such, is used to design the proposed procedure. A case study demonstrates the usefulness of the proposed algorithm to maximize the utility (or to reduce the electricity bill) of a consumer that integrates the proposed procedure in its EMS.

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Antonio J. Conejo (F'04) received the M.S. degree from Massachusetts Institute of Technology, Cambridge, in 1987 and the Ph.D. degree from the Royal Institute of Technology, Stockholm, Sweden in 1990.

He is currently a full Professor at the Universidad de Castilla—La Mancha, Ciudad Real, Spain. His research interests include control, operations, planning, and economics of electric energy systems, as well as statistics and optimization theory and its applications.



Juan M. Morales (S'07) received the Ingeniero Industrial degree from the Universidad de Málaga, Spain, in 2006. He is currently working toward the Ph.D. degree at the Universidad de Castilla-La Mancha, Spain.

His research interests are in the fields of power systems economics, reliability, stochastic programming, and electricity markets.



Luis Baringo (S'09) received the Ingeniero Industrial degree from the Universidad de Castilla—La Mancha, Spain, in 2009. He is currently working toward the Ph.D. degree at the Universidad de Castilla—La Mancha.

His research interests are in the fields of planning and economics of electric energy systems, as well as optimization, stochastic programming, and electricity markets.